אינטגרלים למכינה

1. (א) $\int\left(2x+3e^{x}-x\sqrt{x}\right)dx=x^{2}+3e^{x}-\frac{2}{5}x^{2}\sqrt{x}+C$,

(ב) $\int\left(5\cos x+3^{x}+\frac{1}{2\sqrt[3]{x}}\right)dx=5\sin x+\frac{3^{x}}{\ln 3}+\frac{3}{4}\sqrt[3]{x^{2}}+C$,

(ג) $\int\frac{(18+2x^{2})\sin x+5}{x^{2}+9}dx=\int\left(2\sin x+\frac{5}{x^{2}+9}\right)dx=-2\cos x+\frac{5}{3}\arctan\frac{x}{3}+C$,

(ד) $\int\frac{\left(\sqrt{x}+2\right)^{2}}{\sqrt{x}}dx=\int\left(\sqrt{x}+4+\frac{4}{\sqrt{x}}\right)dx=\frac{2}{3}\sqrt{x^{3}}+4x+8\sqrt{x}+C$,

(ה) $\int\frac{3-\cos^{2}x}{\cos^{2}x}dx=\int\left(\frac{3}{\cos^{2}x}-1\right)dx=3\tan x-x+C$,

(ו) $\int\sqrt[3]{5-6x}dx=-\frac{1}{8}\sqrt[3]{(5-6x)^{4}}+C$,

(ז) $\int\frac{2x^{3}+8x-5}{4+x^{2}}dx=\int\left(2x-\frac{5}{4+x^{2}}\right)dx=x^{2}-\frac{5}{2}\arctan\frac{x}{2}+C$,

(ח) $\int 3^{2-3x}dx=-\frac{3^{2-3x}}{3\ln 3}+C$,

(ט) $\int\frac{dx}{1+2x^{2}}=\frac{1}{2}\int\frac{dx}{\frac{1}{2}+x^{2}}=\frac{1}{\sqrt{2}}\arctan\left(x\sqrt{2}\right)+C$,

(י) $\int\left(\frac{2}{9+x^{2}}+\frac{5}{x^{2}}\right)dx=\frac{2}{3}\arctan\frac{x}{3}-\frac{5}{x}+C$,

(יא) $\int\frac{dx}{\sqrt{36-3x^{2}}}=\frac{1}{\sqrt{3}}\int\frac{dx}{\sqrt{12-x^{2}}}=\frac{1}{\sqrt{3}}\arcsin\frac{x}{\sqrt{12}}+C$,

2. (א) $\int e^{-2x+1}dx=-\frac{1}{2}e^{-2x+1}+C$,

(ב) $\int\frac{dx}{1-3x}=-\frac{1}{3}\ln\left|1-3x\right|+C$,

(ג) $\int\frac{\tan^{2}x+3x\sin^{2}x}{\sin^{2}x}dx=\int\left(\frac{1}{\cos^{2}x}+3x\right)dx=\tan x+\frac{3}{2}x^{2}+C$,

(ד) $\int\frac{dx}{9-4x^{2}}=\frac{1}{4}\int\frac{dx}{\frac{9}{4}-x^{2}}=\frac{1}{4}\frac{1}{3}\ln\left|\frac{\frac{3}{2}+x}{\frac{3}{2}-x}\right|+C=\frac{1}{12}\ln\left|\frac{3+2x}{3-2x}\right|+C$,

(ה) $\int \frac{1+x+3x^2}{\sqrt{x}}dx = \int\left(\frac{1}{\sqrt{x}}+\sqrt{x}+3x\sqrt{x}\right)dx = 2\sqrt{x}+\frac{2}{3}x\sqrt{x}+\frac{6}{5}x^2\sqrt{x}+C$,

(ו) $\int\left(e^{5x}-\frac{2x^2+5}{1+x^2}\right)dx = \int\left(e^{5x}-2-\frac{3}{1+x^2}\right)dx = \frac{1}{5}e^{5x}-2x-3\arctan x+C$,

(ז) $\int \frac{dx}{(5-3x)^2} = \frac{1}{3(5-3x)}+C$,

(ח) $\int \frac{dx}{3-16x^2} = \frac{1}{16}\int \frac{dx}{\frac{3}{16}-x^2} = \frac{1}{8\sqrt{3}}\ln\left|\frac{\frac{\sqrt{3}}{4}+x}{\frac{\sqrt{3}}{4}-x}\right|+C = \frac{1}{8\sqrt{3}}\ln\left|\frac{\sqrt{3}+4x}{\sqrt{3}-4x}\right|+C$,

(ט) $\int \frac{dx}{\sqrt{4-3x^2}} = \frac{1}{\sqrt{3}}\int \frac{dx}{\sqrt{\frac{4}{3}-x^2}} = \frac{1}{\sqrt{3}}\arcsin\frac{x\sqrt{3}}{2}+C$,

(י) $\int \frac{dx}{\sqrt{1-(4x+2)^2}} = \frac{1}{4}\arcsin(4x+2)+C$,

(יא) $\int \frac{dx}{1-5x^2} = \frac{1}{5}\int \frac{dx}{\frac{1}{5}-x^2} = \frac{\sqrt{5}}{2}\ln\left|\frac{\frac{1}{\sqrt{5}}+x}{\frac{1}{\sqrt{5}}-x}\right|+C = \frac{\sqrt{5}}{2}\ln\left|\frac{1+x\sqrt{5}}{1-x\sqrt{5}}\right|+C$.

3. שיטת ההצבה.

(א) $\int \frac{4x^3-1}{x^4-x+5}dx = \left|\begin{matrix} t = x^4-x+5 \\ dt = (4x^3-1)dx \end{matrix}\right| = \int\frac{dt}{t} = \ln|t|+C = \ln\left|x^4-x+5\right|+C$,

(ב) $\int \frac{\sqrt[4]{\ln x}}{x}dx = \left|\begin{matrix} t = \ln x \\ dt = \frac{dx}{x} \end{matrix}\right| = \int\sqrt[4]{t}dt = \frac{4}{5}\sqrt[4]{t^5}+C = \frac{4}{5}\sqrt[4]{\ln^5 x}+C$,

(ג) $\int \frac{3xdx}{(x^2+4)^3} = \left|\begin{matrix} t = x^2+4 \\ dt = 2xdx \end{matrix}\right| = \frac{3}{2}\int\frac{dt}{t^3} = -\frac{3}{4t^2}+C = -\frac{3}{4(x^2+4)^2}+C$,

(ד) $\int \frac{e^{\tan x}}{\cos^2 x}dx = \left|\begin{matrix} t = \tan x \\ dt = \frac{dx}{\cos^2 x} \end{matrix}\right| = \int e^t dt = e^t+C = e^{\tan x}+C$,

(ה) $\int \frac{\cos xdx}{\sqrt{\sin^2 x+3}} = \left|\begin{matrix} t = \sin x \\ dt = \cos xdx \end{matrix}\right| = \int\frac{dt}{\sqrt{t^2+3}} = \ln\left|t+\sqrt{t^2+3}\right|+C =$

$= \ln\left|\sin x+\sqrt{\sin^2 x+3}\right|+C$

(ו) $\int \frac{\sqrt{\arctan x}}{1+x^2}dx = \left|\begin{matrix} t = \arctan x \\ dt = \frac{dx}{1+x^2} \end{matrix}\right| = \int\sqrt{t}dt = \frac{2}{3}\sqrt{t^3}+C = \frac{2}{3}\sqrt{\arctan^3 x}+C$,

(ז) $\int \sqrt[3]{2+7\sin x}\cos x dx = \left|\begin{matrix} t = 2+7\sin x \\ dt = 7\cos x dx \end{matrix}\right| = \frac{1}{7}\int \sqrt[3]{t}dt = \frac{3}{28}\sqrt[3]{t^4} + C = \frac{3}{28}\sqrt[3]{(2+7\sin x)^4} + C$,

(ח) $\int \frac{1}{x^2}\cos\frac{1}{x}dx = \left|\begin{matrix} t = \frac{1}{x} \\ dt = -\frac{dx}{x^2} \end{matrix}\right| = -\int \cos t dt = -\sin t + C = -\sin\frac{1}{x} + C$,

(ט) $\int \frac{\ln x + 2}{2x}dx = \left|\begin{matrix} t = \ln x + 2 \\ dt = \frac{dx}{x} \end{matrix}\right| = \frac{1}{2}\int tdt = \frac{1}{4}t^2 + C = \frac{1}{4}(\ln x + 2)^2 + C$,

(י) $\int \frac{\cos 5x}{\sin^2 5x + 1}dx = \left|\begin{matrix} t = \sin 5x \\ dt = 5\cos 5x dx \end{matrix}\right| = \frac{1}{5}\int \frac{dt}{t^2+1} = \frac{1}{5}\arctan t + C = \frac{1}{5}\arctan(\sin 5x) + C$,

(יא) $\int \frac{dx}{\sin^2 x\sqrt{3-\cot x}} = \left|\begin{matrix} t = 3-\cot x \\ dt = \frac{dx}{\sin^2 x} \end{matrix}\right| = \int \frac{dt}{\sqrt{t}} = 2\sqrt{t} + C = 2\sqrt{3-\cot x} + C$.

4. **אינטגרציה של פונקציות רציונאליות**

(א) $\int \frac{10dx}{15+4x} = \frac{5}{2}\ln|15+4x| + C$,

(ב) $\int \frac{dx}{4-7x} = -\frac{1}{7}\ln|4-7x| + C$,

(ג) $\int \frac{dx}{(x+15)^{20}} = -\frac{1}{19(x+15)^{19}} + C$,

(ד) $\int \frac{dx}{(5-2x)^8} = \frac{1}{14(5-2x)^7} + C$,

(ה) $\int \frac{xdx}{x^2-7x+20} = \left|\begin{matrix} x^2-7x+20 = \left(x-\frac{7}{2}\right)^2 + \frac{31}{4} \\ t = x-\frac{7}{2},\ x = t+\frac{7}{2}, dt = dx \end{matrix}\right| = \int \frac{t+\frac{7}{2}}{t^2+\frac{31}{4}}dt = \int \frac{tdt}{t^2+\frac{31}{4}} +$

$$+\frac{7}{2}\int \frac{dt}{t^2+\frac{31}{4}} = \frac{1}{2}\ln\left(t^2+\frac{31}{4}\right) + \frac{7}{\sqrt{31}}\arctan\frac{2t}{\sqrt{31}} + C =$$

$$= \frac{1}{2}\ln(x^2-7x+20) + \frac{7}{\sqrt{31}}\arctan\frac{2x-7}{\sqrt{31}} + C$$

(ו) $\int \frac{x}{2x^2+2x+5}dx = \frac{1}{2}\int \frac{x}{x^2+x+5/2}dx = \left|\begin{matrix} t = x+\frac{1}{2}, dx = dt \\ x = t-\frac{1}{2} \end{matrix}\right| = \frac{1}{2}\int \frac{t-\frac{1}{2}}{t^2+\frac{9}{4}}dt =$

$$= \frac{1}{2}\int \frac{tdt}{t^2+\frac{9}{4}} - \frac{1}{4}\int \frac{dt}{t^2+\frac{9}{4}} = \frac{1}{4}\ln\left(t^2+\frac{9}{4}\right) - \frac{1}{4}\frac{2}{3}\arctan\frac{2t}{3} + C =$$

$$= \frac{1}{4}\ln\left(x^2 + x + \frac{5}{2}\right) - \frac{1}{6}\arctan\frac{2x+1}{3} + C$$

$$,\int \frac{x+1}{5x^2+2x+1}dx = \frac{1}{10}\ln\left(5x^2+2x+1\right) + \frac{1}{5}\arctan\frac{5x+1}{2} + C \quad (ז)$$

$$\int \frac{2x+1}{x^2-4x+5}dx = \left|\begin{matrix} x^2-4x+5 = (x-2)^2+1 \\ t = x-2, x = t+2, dx = dt \end{matrix}\right| = \int \frac{2t+5}{t^2+1}dt = \int \frac{2tdt}{t^2+1} + 5\int \frac{dt}{t^2+1} = \quad (ח)$$

$$= \ln(t^2+1) + 5\arctan t + C = \ln(x^2-4x+5) + 5\arctan(x-2) + C$$

$$\int \frac{dx}{2x^2-2x+3} = \frac{1}{2}\int \frac{dx}{x^2-x+\frac{3}{2}} = \frac{1}{2}\int \frac{dx}{\left(x-\frac{1}{2}\right)^2+\frac{5}{4}} = \frac{1}{2}\frac{2}{\sqrt{5}}\arctan\frac{2\left(x-\frac{1}{2}\right)}{\sqrt{5}} + C = \quad (ט)$$

$$= \frac{1}{\sqrt{5}}\arctan\frac{2x-1}{\sqrt{5}} + C$$