בפתרון שאלה זו ניעזר במשפט הבא: אם $a_n \to \infty$ ו־$x \in \mathbb{R}$, אז $\lim_{n\to\infty}\left(1+\frac{x}{a_n}\right)^{a_n} = e^x$.

(א)

$$\lim_{n\to\infty}\left(1+\frac{7}{n}\right)^{n+1} = \lim_{n\to\infty}\left(1+\frac{7}{n}\right)^{n}\cdot\left(1+\frac{7}{n}\right) = e^7\cdot 1 = e^7$$

(ב)

$$\lim_{n\to\infty}\left(\frac{n+3}{n+1}\right)^{2n} = \lim_{n\to\infty}\left(\frac{n+1+2}{n+1}\right)^{2n} = \lim_{n\to\infty}\left(1+\frac{2}{n+1}\right)^{2n} =$$

$$= \lim_{n\to\infty}\left[\left(1+\frac{2}{n+1}\right)^{n+1}\right]^2\cdot\left(1+\frac{2}{n+1}\right)^{-2} = (e^2)^2\cdot 1^{-2} = e^4$$

(ג)

$$\lim_{n\to\infty}\left(\frac{7+3n}{9+3n}\right)^{n} = \lim_{n\to\infty}\left(\frac{9+3n-2}{9+3n}\right)^{n} = \lim_{n\to\infty}\left(1-\frac{2}{9+3n}\right)^{n} =$$

$$= \lim_{n\to\infty}\left[\left(1-\frac{2}{9+3n}\right)^{9+3n}\right]^{\frac{1}{3}}\cdot\left(1-\frac{2}{9+3n}\right)^{-3} = \left(e^{-2}\right)^{\frac{1}{3}}\cdot 1^{-3} = e^{-\frac{2}{3}}$$

(ד)

$$\lim_{n\to\infty}\left(\frac{n^2-2}{n^2-3}\right)^{4n^2-1} = \lim_{n\to\infty}\left(\frac{n^2-3+1}{n^2-3}\right)^{4n^2-1} = \lim_{n\to\infty}\left(1+\frac{1}{n^2-3}\right)^{4n^2-1} =$$

$$= \lim_{n\to\infty}\left[\left(1+\frac{1}{n^2-3}\right)^{n^2-3}\right]^4\cdot\left(1+\frac{1}{n^2-3}\right)^{11} = e^4\cdot 1^{11} = e^4$$