## הוכחה (באדיבות בנימין)

$$r=1$$

$$df\_{a}\left(h\right)=\sum\_{j=1}^{n}\frac{∂f}{∂x\_{j}}\left(a\right)h\_{j}$$

כלומר כאן $\left|α\right|=r=1$ $⇐$ $α=δ\_{j}=\left(0,0,..,1,…,0\right)$ כלומר $α!=1$

$$\sum\_{j=1}^{n}\frac{∂f}{∂x\_{j}}\left(a\right)h\_{j}=\sum\_{j=1}^{n}\frac{1!}{1!}Df^{δ\_{j}}\left(a\right)h^{δ\_{j}}$$

נניח ש:

$$\frac{d^{r}}{dt^{r}}f\left(a+th\right)|\_{t=0}=\sum\_{\left|α\right|=r}^{}\frac{r!}{α!}D^{α}f\left(a\right)h^{α}$$

אם הדבר נכון לכל $a$, נחליף ל$a+sh$

$$\frac{d^{r}}{dt^{r}}f\left(a+sh+th\right)|\_{t=0}=\sum\_{\left|α\right|=r}^{}\frac{r!}{α!}D^{α}f\left(a+sh\right)h^{α}$$

$$\frac{d^{r}}{dt^{r}}f\left(a+h\left(s+t\right)\right)|\_{t=0}=\left\{u=t+s\right\}=\frac{d^{r}}{du^{r}}f\left(a+hu\right)|\_{t=0}=\sum\_{\left|α\right|=r}^{}\frac{r!}{α!}D^{α}f^{α}\left(a\right)h^{a}$$

$$\frac{d}{ds}:\left(\*\right)\frac{d^{r+1}}{ds^{r+1}}f\left(a+sh\right)=\sum\_{\left|α\right|=1}^{}\frac{r!}{α!}\frac{d}{ds}D^{α}f\left(a+sh\right)h^{α}=\sum\_{\left|α\right|=1}^{}\frac{r!}{α!}\left(\sum\_{j=1}^{n}\frac{∂}{∂x\_{j}}D^{α}f\left(a+sh\right)h\_{j}\right)h^{α}$$

נגדיר $δ\_{j}=\left(0,…,1,.,,0\right)$

$$h\_{j}h^{α}=h^{α+δ\_{j}}⇒\frac{∂}{∂x\_{j}}D^{α}f=D^{α+δ\_{j}}f$$

$$\left(\*\right)=\sum\_{\left|α\right|=r}^{}\frac{r!}{α!}\sum\_{j=1}^{n}D^{α+δ\_{j}}f\left(a+sh\right)h^{α+δ\_{j}}$$

נסמן $β≔α+δ\_{j}$

$$\left|β\right|=r+1⇒\frac{β!}{β\_{j}}=α!⇒$$

$$\left(\*\right)=\sum\_{j=1}^{n}\sum\_{\left|β\right|=r+1}^{}\frac{r!β\_{j}}{β!}D^{β}f\left(a+sh\right)h^{β}=\sum\_{\left|β\right|=r+1}^{}\frac{r!}{β!}\sum\_{j=1}^{n}β\_{j}D^{β}f\left(a+sh\right)h^{β}$$

$$β\_{1}+…+β\_{n}=\left|β\right|=r+1⇒\sum\_{j=1}^{n}β\_{j}=r+1$$

$$⇒\left(\*\right)=\sum\_{\left|β\right|=r+1}^{}\frac{r!\left(r+1\right)}{β!}D^{β}f\left(a+sh\right)h^{β}$$

$$s=0⇒\frac{d^{r+1}}{ds^{r+1}}f\left(a+sh\right)|\_{s=0}=\sum\_{\left|β\right|=r+1}^{}\frac{\left(r+1\right)!}{β!}D^{β}f\left(a\right)h^{β}$$

ולכן לפי אינדוקציה כדרוש.