LINEAR ALGEBRA
EXERCISE 2

3.15:
a. $f_A(x) = x^3 - 15x^2 + 30x - 48$.
$f_{2A}(x) = x^3 - 10x^2 + 120x - 384$.
b. Prove that $f_{\alpha A}(x) = \alpha^n f_A(\frac{x}{2})$.
Solution: We divide each row of A by α.

$$\begin{matrix} \frac{x}{\alpha} - a_{11} & -a_{12} & -a_{13} & & -a_{1n} \\ -a_{21} & \frac{x}{\alpha} - a_{22} & -a_{23} & & -a_{2n} \\ & & & & \\ -a_{n1} & -a_{n2} & & & \frac{x}{\alpha} - a_{nn} \end{matrix}$$

So we have to multiply the determinant by α^n.
$f_{2A}(x) = 2^3[(\frac{x}{2})^3 - 5(\frac{x}{2})^2 + 30(\frac{x}{2}) - 48]$.

3.17: A is the matrix:

$$\begin{matrix} 0 & 1 & 0 \\ 5 & 10 & -4 \\ 16 & 32 & -12 \end{matrix}$$

$f_A(x) = x(x^2+2x-120+128)+(-5x-60+64) = x^3+2x^2+3x+4$. So By Cayly Hamilton's Theorem $A^3+2A^2+3A+4I = 0$. So $\frac{1}{-4}A(A^2+2A+3I) = I$. So $A^{-1} = \frac{1}{-4}(A^2 + 2A + 3I)$. But A^2 is equal to:

$$\begin{matrix} 5 & 10 & -4 \\ -14 & -23 & 8 \\ -32 & -48 & 16 \end{matrix}$$

So $-4A^{-1}$ is equal to:

$$\begin{matrix} 8 & 12 & -4 \\ -4 & 0 & 0 \\ 0 & 16 & -5 \end{matrix}$$