<u>פתרון תרגיל 3</u>

1) א) נרשום

$$\int_{-a}^{a} f(x)\,dx = \int_{-a}^{0} f(x)\,dx + \int_{0}^{a} f(x)\,dx$$

באינטגרל הראשון נציב $x=-t$ ונקבל

$$\int_{-a}^{0} f(x)\,dx = \left\{\begin{matrix} x=-t \\ dx=-dt \end{matrix}\right\} = \int_{a}^{0} f(-t)(-dt) =$$

$$= -\int_{a}^{0} f(t)(-dt) = \int_{a}^{0} f(t)\,dt = -\int_{0}^{a} f(t)\,dt$$

f אי-זוגית

x ו-t משתנים מדומים בקטע $[0,a]$ לכן

$$\int_{0}^{a} f(x)\,dx = \int_{0}^{a} f(t)\,dt$$

לכן

$$\int_{-a}^{a} f(x)\,dx = -\int_{0}^{a} f(t)\,dt + \int_{0}^{a} f(x)\,dx = 0$$

ב) קל לראות ש- $f(x)=\dfrac{x^5}{3+\cos x}$ אי-זוגית.

שהרי $f(-x)=\dfrac{-x^5}{3+\cos(-x)} = -\dfrac{x^5}{3+\cos x} = -f(x)$

לכן לפי סעי' א' $\displaystyle\int_{-1}^{1} \frac{x^5}{3+\cos x}\,dx = 0$.

2) א)

$$\int \frac{dx}{x\sqrt{1+\ln x}} = \left\{\begin{matrix} \ln x = t \\ \frac{1}{x}dx = dt \end{matrix}\right\} = \int \frac{dt}{\sqrt{1+t}} =$$

$$= 2\sqrt{1+t} + c = 2\sqrt{1+\ln x} + c$$

$$\int \frac{x\,dx}{1+\sqrt{x}} = \left\{ \begin{array}{l} 1+\sqrt{x} = t \\ \frac{1}{2\sqrt{x}}\,dx = dt \end{array} \middle| \, dx = 2(t-1)\,dt \right\}$$

$$= \int \frac{2(t-1)^2(t-1)}{t}\,dt = \int \frac{2(t-1)^3}{t}\,dt =$$

$$= 2\int \frac{t^3-3t^2+3t-1}{t}\,dt = 2\int \left(t^2-3t+3-\frac{1}{t}\right)dt$$

$$= 2\left[\frac{t^3}{3} - \frac{3t^2}{2} + 3t - \ln|t|\right] + c =$$

$$= \frac{2}{3}(1+\sqrt{x})^3 - \frac{3}{2}(1+\sqrt{x})^2 + \frac{3}{2}(1+\sqrt{x}) - 2\ln|1+\sqrt{x}| + c$$

$$\int \frac{dx}{x^2+4x+5} = \int \frac{dx}{(x+2)^2+1} = \left\{ \begin{array}{l} x+2 = t \\ dx = dt \end{array} \right\}$$

$$= \int \frac{dt}{t^2+1} = \arctan t + c = \arctan(x+2) + c$$

$$\int x(1-x)^{100}\,dx = \left\{ \begin{array}{l} t = 1-x \\ dt = -dx \end{array} \right\} = \int (1-t)\,t^{100}\,dt$$

$$= \int \left(t^{100} - t^{101}\right)dt = \frac{t^{101}}{101} - \frac{t^{102}}{102} + c =$$

$$= \frac{(1-x)^{101}}{101} - \frac{(1-x)^{102}}{102} + c$$

$$\int \frac{2+x^2}{\sqrt{4-x^2}}\,dx = \int \frac{6-(4-x^2)}{\sqrt{4-x^2}}\,dx = \int \frac{6}{\sqrt{4-x^2}} - \int \frac{(4-x^2)}{\sqrt{4-x^2}}\,dx$$

$$= \int \frac{6}{\sqrt{4-x^2}}\,dx - \int \sqrt{4-x^2}\,dx$$

פתרון ל אינטגרל,

$$6\int\frac{dx}{\sqrt{4-x^2}} = 6\int\frac{dx}{\sqrt{4\left(1-\frac{x^2}{4}\right)}} = 6\int\frac{dx}{2\sqrt{1-\left(\frac{x}{2}\right)^2}} =$$

$$=3\int\frac{dx}{\sqrt{1-\left(\frac{x}{2}\right)^2}} = \left\{\begin{array}{l} t=\frac{x}{2} \\ dt=\frac{1}{2}dx \end{array}\right\} = 3\int\frac{2\,dt}{\sqrt{1-t^2}} =$$

$$=6\int\frac{dt}{\sqrt{1-t^2}} = 6\arcsin t + c = 6\arcsin\left(\frac{x}{2}\right) + c$$

$$\int\sqrt{4-x^2}\,dx = \left\{\begin{array}{l} x=2\sin t \\ dx=2\cos t\,dt \end{array}\right\} =$$

$$=\int\sqrt{4-4\sin^2 t}\;2\cos t\,dt = \int\sqrt{4\cos^2 t}\cdot 2\cos t\,dt$$

$$=4\int\cos t\cdot\cos t\,dt = 4\int\frac{1+\cos 2t}{2}\,dt$$

$$=2\int(1+\cos 2t)\,dt = 2t+\sin 2t + c$$

$$=2\arcsin\left(\frac{x}{2}\right)+\sin\left(2\cdot\arcsin\left(\frac{x}{2}\right)\right)+c$$

$$\int\frac{(e^{2x}+2)(e^{3x}+3)}{e^{x}}\,dx = \int\frac{e^{5x}+3e^{2x}+2e^{3x}+6}{e^{x}}\,dx \quad (!$$

$$=\int\left(e^{4x}+3e^{x}+2e^{2x}+6e^{-x}\right)dx = \frac{e^{4x}}{4}+3e^{x}+e^{2x}-6e^{-x}+c$$

$$\int\frac{x^2}{x^6+4}\,dx = \int\frac{x^2}{(x^3)^2+4}\,dx = \left\{\begin{array}{l} t=x^3 \\ dt=3x^2\,dx \end{array}\right\}$$

$$=\int\frac{\frac{dt}{3}}{t^2+4} = \frac{1}{3}\int\frac{dt}{t^2+4} = \frac{1}{3}\int\frac{dt}{4\left(\left(\frac{t}{2}\right)^2+1\right)}$$

$$= \frac{1}{12} \int \frac{dt}{\left(\frac{t}{2}\right)^2 + 1} = \left\{ \begin{array}{l} u = \frac{t}{2} \\ du = \frac{1}{2} dt \end{array} \right\} =$$

$$= \frac{1}{12} \int \frac{2\,du}{u^2+1} = \frac{1}{6} \int \frac{du}{u^2+1} = \frac{1}{6} \arctan u + c$$

$$= \frac{1}{6} \arctan\left(\frac{x^3}{2}\right) + c$$

$$\int \frac{\arcsin x}{\sqrt{x+1}}\,dx = \left\{ \begin{array}{l|l} u = \arcsin x & u' = \frac{1}{\sqrt{1-x^2}} \\ v' = \frac{1}{\sqrt{x+1}} & v = 2\sqrt{x+1} \end{array} \right\} =$$

$$= 2 \operatorname{arsin}(x) \cdot \sqrt{x+1} - \int \frac{2\sqrt{x+1}}{\sqrt{1-x^2}}\,dx =$$

$$= 2\sqrt{x+1} \arcsin x - 2 \int \sqrt{\frac{x+1}{(1-x)(1+x)}}\,dx =$$

$$= 2\sqrt{x+1} \arcsin x - 2 \int \frac{1}{\sqrt{1-x}}\,dx =$$

$$= 2\sqrt{x+1} \arcsin x + 2 \sqrt{1-x} + c$$

$$\int \frac{e^{2x}}{\sqrt[4]{e^x+1}}\,dx = \left\{ \begin{array}{l} t = e^x \\ dt = e^x dx \end{array} \right\} = \int \frac{t\,dt}{\sqrt[4]{1+t}}$$

$$= \int \frac{t+1-1}{\sqrt[4]{t+1}}\,dt = \int \frac{t+1}{\sqrt[4]{t+1}}\,dt - \int \frac{1}{\sqrt[4]{t+1}}\,dt$$

$$= \int (t+1)^{\frac{3}{4}}\,dt - \int (t+1)^{-\frac{1}{4}}\,dt = \frac{(t+1)^{\frac{7}{4}}}{\frac{7}{4}} - \frac{(t+1)^{\frac{3}{4}}}{\frac{3}{4}} + c$$

$$= \frac{4}{7}(e^x+1)^{\frac{7}{4}} - \frac{4}{3}(e^x+1)^{\frac{3}{4}} + c$$

$$\int \cos(\ln x)\,dx = \left\{ \begin{array}{l|l} u=\cos(\ln x) & u'=\frac{-\sin(\ln x)}{x} \\ v'=1 & v=x \end{array} \right\} =$$

$$= x\cos(\ln x) - \int \frac{-\sin(\ln x)}{x}\cdot x\,dx =$$

$$= x\cos(\ln x) + \int \sin(\ln x)\,dx = \left\{ \begin{array}{l|l} u=\sin(\ln x) & u'=\frac{\cos(\ln x)}{x} \\ v'=1 & v=x \end{array} \right\} =$$

$$= x\cos(\ln x) + x\sin(\ln x) - \int \cos(\ln x)\,dx$$

$$2\int \cos(\ln x)\,dx = x\cos(\ln x) + x\sin(\ln x)$$

$$\int \cos(\ln x)\,dx = \frac{x\cos(\ln(x)) + x\sin(\ln x)}{2} + C$$